4.

Case i: $a > b > c$

$Min(a, min(b, c)) = min(min(a,b),c)$
$Min(a,c)=Min(b,c)$
$c=c$

Case ii: $a>c>b$
$Min(a, min(b, c)) = min(min(a,b),c)$
$Min(a,b)=Min(b,c)$
$b=b$

Case iii: $c>b>a$
$Min(a, min(b, c)) = min(min(a,b),c)$
$Min(a,b)=Min(a,c)$
$a=a$

Case iv: WLOG when ‘a’ is lowest the result of the first step on the right side would be the same regardless of “min(a,b)” regardless of its comparison to b or c. The result of the second step on the right side would be the same regardless of whether ‘a’ were compared to b or c as it is the absolute lowest. In the second step on the left side, the result will be the same independent of the evaluation of the first step on the left side as ‘a’ is once again the absolute lowest. This shows that, with ‘a’ being the lowest of the numbers, the relative sizes of ‘b’ and ‘c’ do not affect the outcome.

Case v: WLOG using the exact same steps in Case iv, we conclude that with ‘b’ being the lowest of the numbers, the relative sizes of ‘a’ and ‘c’ do not affect the outcome.

Case vi: WLOG using the exact same steps in Case iv, we conclude that with ‘c’ being the lowest of the numbers, the relative sizes of ‘a’ and ‘b’ do not affect the outcome.

10. For a number, r, to be a perfect square, there must exist some integer, s, such that $s^2 = r$. As the only r, such that r is a perfect square and $r+1$ is a perfect square is $r=0$, and the difference between these two numbers is 1, we can conclude that one of them must not be a perfect square. This is a nonconstructive proof.

22. $(x-1/x)^2 >= 0$
$x^2-2/x+1/x^2 >=0$
$x^2+1/x^2 >= 2/x$
$x^3+1/x >= 2$

Case i: $0<x<$

24. Arithmetic mean is denoted by AM(x,y) and Quadratic mean is denoted by QM(x,y)
$AM(1,1) = 1$
$QM(1,1)= \sqrt{2}$
$AM(1,2) = 1.5$
$QM(1,2) = 1.581$
$AM(5,5) = 5$
$QM(5,5) = 5$
$AM(5,6) = 5.5$
$QM(5,6) = 5.52$

Conjecture: for any two real numbers, x and y, $QM(x,y) \geq AM(x,y)$

$(x+y)/2 \leq \sqrt{(x^2 + y^2)/2}$
$((x+y)^2)/4 \leq (x^2+y^2)/2$
$((x+y)^2)/2 \leq x^2+y^2$
$(x^2+2xy+y^2)/2 \leq x^2+y^2$
$x^2+2xy+y^2 \leq 2x^2 + 2y^2$
$2xy \leq x^2+y^2$
$2 \leq x/y+y/x$

36. A number is rational if there exist an a,b such that the number is equal to a/b.

44.